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LETTER TO THE EDITOR

The normal current in a thin film of superfluid ^{3}He-A

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Abstract. A thin film of superfluid ^{3}He-A film containing a polar domain wall has a normal current flowing in it even at zero temperature. We calculate this current from the Gorkov–Nambu equations in a way similar to that of Volovik for bulk ^{3}He-A. An earlier calculation by the authors working to first order in gradients yielded a singular normal current (proportional to a delta function). In the present work we show that the singular nature is an artefact of the gradient expansion. An additional length scale in the problem smooths out the delta singularity. We show that the current obtained has its origins in a zeroth Landau level.

Recently the authors studied the mass current associated with a polar domain wall in a thin film of superfluid ^{3}He-A (Pattarini and Waxman 1986 hereafter referred to as I).

In I we implicitly made the following assumptions that we will also make in this work:

(i) the polar domain wall of Ohmi *et al* (1982) exists at zero temperature (it was determined at temperatures close to T_c).

(ii) to a reasonable approximation, the thin film with the domain wall is described by the order parameter (cf Nakahara 1986) (see I for notation).

$$C = 2^{-1/2}(\boldsymbol{\varphi}_1 + \mathrm{i}\boldsymbol{\varphi}_2)\cdot\boldsymbol{p} \tag{1}$$

with

$$\boldsymbol{\varphi}_1 = \hat{\boldsymbol{e}}_x \tag{2}$$

$$\boldsymbol{\varphi}_2 = \tanh(Bx)\hat{\boldsymbol{e}}_y \tag{3}$$

(in this work we denote the wall width by B^{-1}).

In I it was shown that the standard gradient expansion for the 'anomalous' part of the current (associated with a normal fluid component of Volovik (1985)) yielded (equation (15) of I)

$$\boldsymbol{g}_c = (\rho/m)\delta(x)\hat{\boldsymbol{e}}_y. \tag{4}$$

We commented at the time on the curious nature of this result: that it is singular and independent of the wall width B^{-1}. It is the purpose of the present work to show that these features are an *artefact* of the gradient expansion and that equation (4) results from a 'zeroth Landau level' similar to the case of Volovik (1985) in bulk ^{3}He-A.

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The procedure we adopt bears many similarities to that of Volovik (1985) however there are also some differences.

The calculation for the current is in two steps; firstly one solves the Gorkov–Nambu equations

$$(\mathrm{i}\omega - H)\begin{pmatrix} G \\ \bar{F} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

for the Green function G and then the current is obtained from G in the standard way.

The energy eigenvalue equation associated with (5) is

$$\begin{pmatrix} \left(\dfrac{\hat{p}^2}{2m} - \mu\right) & \dfrac{1}{2P_{\mathrm{F}}}\{\hat{p}_j, \Delta_j\} \\ \dfrac{1}{2p_{\mathrm{F}}}\{\hat{p}_j, \Delta_j^*\} & -\left(\dfrac{\hat{p}^2}{2m} - \mu\right) \end{pmatrix}\begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n\begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} \tag{6}$$

in which

$$\boldsymbol{\Delta} = \Delta_0(\boldsymbol{\varphi}_1 + \mathrm{i}\boldsymbol{\varphi}_2) \tag{7}$$

with $\boldsymbol{\varphi}_1$, $\boldsymbol{\varphi}_2$ given by equations (2), (3).

In equation (6) we impose vanishing boundary conditions at $z = 0$ and $z = l$ (l = film thickness) and periodic BC in x, y over a square of side L. A normalised solution is written in the form

$$\begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = \frac{\exp(\mathrm{i}k_y y)}{L^{1/2}}\left(\frac{2}{l}\right)^{1/2} \sin(k_z z)\begin{pmatrix} u_n^0(x) \\ v_n^0(x) \end{pmatrix}. \tag{8}$$

Using (8) in equation (6) and making the approximations (i) of neglecting the terms $\hat{p}_x^2/2m$ and (ii) of linearising $\tanh Bx$ about $x = 0$ ($\tanh Bx \simeq Bx$) we obtain

$$\left(\varepsilon\sigma^3 + \frac{\Delta_0}{p_{\mathrm{F}}}\{\hat{p}_x\sigma^1 - k_y Bx\sigma^2\}\right)\begin{pmatrix} u_n^0 \\ v_n^0 \end{pmatrix} = E_n(\varepsilon)\begin{pmatrix} u_n^0 \\ v_n^0 \end{pmatrix} \tag{10}$$

in which

$$\varepsilon = \frac{k_y^2 + k_z^2}{2m} - \mu \tag{11}$$

and σ^i are the Pauli spin matrices.

Approximation (i) is justified since important values of k_y^2 are $\mathrm{O}(p_{\mathrm{F}}^2)$ and $\langle p_x^2\rangle \sim |k_y|B \ll p_{\mathrm{F}}^2$.

It is straightforward to show that the eigenvalues of (10) are

$$E_n(\varepsilon) = \left(\varepsilon^2 + 2n\left(\frac{\Delta_0}{p_{\mathrm{F}}}\right)^2 |k_y|B\right)^{1/2} \operatorname{sgn}(\varepsilon) \qquad n = 0, 1, 2, \ldots \tag{12}$$

with eigen-functions

$$\begin{bmatrix} u_n^0(x) \\ v_n^0(x) \end{bmatrix} = \begin{bmatrix} \alpha_n f_n(x) \\ \beta_n f_{n-1}(x) \end{bmatrix} \qquad k_y > 0 \tag{13}$$

$$= \begin{bmatrix} \alpha_n f_{n-1}(x) \\ \beta_n f_n(x) \end{bmatrix} \qquad k_y < 0. \tag{14}$$

In the above, $f_n(x)$ are normalised energy eigen-functions of a harmonic oscillator of mass $\frac{1}{2}(p_F/\Delta_0)^2$ and angular frequency $2(\Delta_0/p_F)^2|k_y|B$ and

$$|\alpha_n|^2 + |\beta_n|^2 = 1 \tag{15}$$

$$|\alpha_n|^2 = \tfrac{1}{2}(1 + \varepsilon/E_n). \tag{16}$$

From the above, it follows that the mass current at $T = 0$ is

$$g(r) = \sum_{k_y, k_z, n} |\alpha_n|^2 k_y \hat{e}_y \frac{2}{Ll} \sin^2(k_z z) |f_n(x)|^2 \tag{17}$$

the step function $\theta(-E_n)$ arising from the zero-temperature limit of the Fermi function.

The contribution to the current from states with $n = 0$ (the zeroth Landau level) only arises from positive k_y since by equations (12), (15) and (16) the eigen-function in equation (14) vanishes identically (for $n = 0$).

The spectrum of the zeroth Landau level exhibits asymmetry with respect to k_y and from equation (12) we have

$$E_0(\varepsilon) = \varepsilon = \frac{k_y^2 + k_z^2}{2m} - \mu. \tag{18}$$

This level has (in the approximations considered) a gapless spectrum and the current associated with this level is, accordingly, normal.

We can calculate the normal mass current at $T = 0$ (associated with the zeroth Landau level) from equation (17) by approximating both sums by integrals (valid for $p_F l \gg 1$), replacing $\sin^2(k_z z)$ by $\frac{1}{2}$ (valid for distances $\gtrsim p_F^{-1}$ from the film edges), using equations (16), (18) and

$$|f_0(x)|^2 = \left(\frac{|k_y|B}{\pi}\right)^{1/2} \exp(-|k_y|Bx^2) \tag{19}$$

we obtain

$$g_c(r) = \hat{e}_y \int_0^\infty \frac{dk_y}{\pi} \int_0^\infty \frac{dk_z}{2\pi}\, \theta(p_F^2 - k_y^2 - k_z^2) k_y \left(\frac{k_y B}{\pi}\right)^{1/2} \exp(-k_y B x^2). \tag{20}$$

On integrating over k_z and transforming to a dimensionless variable $u = k_y/p_F$ we obtain (a factor of two has been included for spin and $\rho \equiv m p_F^3/3\pi^2$)

$$g_c(r) = (\rho/m) h(x) \hat{e}_y \tag{21}$$

where

$$h(x) = \frac{3}{(\pi)^{1/2}} \frac{1}{\eta} \int_0^1 du\, u^{3/2} (1 - u^2)^{1/2} \exp(-ux^2/\eta^2) \tag{22}$$

and

$$\eta \equiv (p_F B)^{-1/2}. \tag{23}$$

It is easily shown that $h(x)$ has area unity, width $\sim\eta$ and thus on length scales large compared with η has all the properties of a δ function. Equation (21) exactly coincides with equation (4) in the limit $\eta \to 0$. The result of equation (21) is physically preferable to that of equation (4) since it is non-singular and does (as one would expect) depend on the wall width.

It is interesting to consider the length scale $\eta = (p_F B)^{-1/2}$ that has appeared in the problem.

Taking $B^{-1} \sim 10\xi_0$, say, $p_F \sim a^{-1}$, a the inter-particle spacing we have

$$\eta \sim (a\xi_0)^{1/2}. \tag{24}$$

It is clear that from the viewpoint of the gradient expansion η is treated as zero rather than its small (on the scale of ξ_0) but finite value.

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