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# Appendix B from P. Nouvellet et al., "Fundamental Insights into the Random Movement of Animals from a Single Distance-Related Statistic"

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## Relationship between the Correlation Function and Mean Square Displacement

In this appendix, we derive the relationship between the correlation function $\Delta(t)$ and the mean square displacement $\sigma^2(t)$. We achieve this by solving the equation that the displacement $\mathbf{R}(t) = (X(t), Y(t))$ obeys, namely, equation (A1). The solution of equation (A1) subject to $\mathbf{R}(0) = (0, 0)$ is $\mathbf{R}(t) = \int_0^t \eta(s)ds$, that is, $X(t) = \int_0^t \eta_X(s)ds$ and $Y(t) = \int_0^t \eta_Y(s)ds$. Using these results, the mean square displacement $\sigma^2(t) = \mathrm{E}\left[X^2(t) + Y^2(t)\right]$ can be expressed as $\mathrm{E}\left[\left(\int_0^t \eta_X(s)ds\right)^2 + \left(\int_0^t \eta_Y(s)ds\right)^2\right]$. Both the $X$- and $Y$-components of the $\eta$ terms make identical contributions to $\sigma^2(t)$. We thus only consider $\mathrm{E}\left[\left(\int_0^t \eta_X(s)ds\right)^2\right]$. Using the definition of the correlation function $\mathrm{E}[\eta_X(s_1)\eta_X(s_2)] = \Delta(s_1 - s_2)$, we obtain $\mathrm{E}\left[\left(\int_0^t \eta_X(s)ds\right)^2\right] = \int_0^t\int_0^t \Delta(s_1 - s_2)ds_1 ds_2$. Thus, the relationship between $\Delta(t)$ and $\sigma^2(t)$ is $\sigma^2(t) = 2\int_0^t\int_0^t \Delta(s_1 - s_2)ds_1 ds_2$. This result can also be written in the simpler form $\sigma^2(t) = 4\int_0^t (t - s)\Delta(s)ds$.